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Electrorotation: a spherical shell model

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Abstract. The spinning of a homogeneous spherical object and of a spherical object surrounded by a thin shell, when placed in various electric field configurations is studied. In static electric fields the electrorotation appears if the electric field strength exceeds a threshold value. In alternating fields this threshold is frequency dependent. The electrorotation spectra for rotating electric fields are analysed emphasising the shell contribution. We also analyse the connection between dielectrophoresis and electrorotation.

1. Introduction

The spinning of small physical objects can occur when they are subjected to static, alternating or rotating electric fields.

The phenomenon originates in the appearence of a non-zero electric torque acting on a suspended object in different field geometries. The polarisation mechanism effective at frequencies up to 10^7 Hz is the interfacial Maxwell–Wagner polarisation. The same polarisation mechanism is responsible for the appearance of the dielectrophoretic force, so these phenomena are strongly interrelated.

The spinning of solid particles in strong electrostatic fields was reported as early as 1896 by Quincke (1896). The phenomenon exhibits a threshold value of the electric field and occurs only if certain conditions concerning the electric properties of the particle and of the external medium are satisfied. Lampa (1906) was the first to state correctly the general condition required for spontaneous rotation. These pioneering works were followed by many theoretical or applied contributions, reviewed in a paper by Jones (1984).

Electrorotation can occur not only in static electric fields but also in alternating electric fields. The phenomenon was observed both for multicell (Teixeira-Pinto *et al* 1960, Holzapfel *et al* 1982) and for single-cell (Pohl and Crane 1971) cases.

The multicell electrorotation was attributed to a dipole–dipole interaction between neighbouring cells (Holzapfel *et al* 1982) which leads to a non-zero time-averaged electric torque.

A theoretical investigation of the single-cell rotation has been reported by one of us (Turcu 1987). A dynamical symmetry breaking mechanism able to describe the appearance of the rotational motion was proposed.

Another possibility for inducing rotation is to use continuous (Arnold and Zimmermann 1982) or pulsed (Mischel *et al* 1982, Glaser *et al* 1983) rotating electric fields. The electrorotation spectra have a relatively sharp maximum at a characteristic frequency and in some cases even a second maximum could appear (Fuhr *et al* 1985). Several theoretical approaches have been proposed in order to explain the experimental

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facts (Arnold and Zimmermann 1982, Hagedorn and Fuhr 1984, Lovelace *et al* 1984, Sauer and Schlögl 1985, Fuhr *et al* 1986).

In the preceding paper (Turcu and Lucaciu 1989) we analysed the dielectrophoretic force acting on a simple spherical object and on a spherical object surrounded by a single shell.

In the present paper we propose a unitary approach to all electrorotation phenomena, giving the complete algebraic expression and some systematic approximations for the electric torque for the same models. The connection between dielectrophoresis and electrorotation is also analysed.

The notation is the same as in the preceding paper.

2. Electrorotation in static homogeneous fields

We shall briefly discuss the spherical model.

The rotational motion occurs if the appearance of a non-zero electric torque

$$\boldsymbol{T}_e = \boldsymbol{P}_{\text{eff}} \times \boldsymbol{E} \tag{1}$$

tends to accelerate any random initial rotation, causing the static steady state to become unstable.

In order to find the expression for the torque we must calculate the effective dipole moment induced in an anticlockwise rotating sphere, for instance by a static homogeneous electric field. In the rotating coordinate system, at rest with respect to the sphere, the field becomes a clockwise rotating vector:

$$\hat{\boldsymbol{E}} = (\boldsymbol{e}_1 - \mathrm{i}\,\boldsymbol{e}_2)\exp(-\mathrm{i}\omega t) \tag{2}$$

where ω is the angular frequency of the sphere and $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ are two orthogonal unit vectors in the rotation plane.

Introducing the dimensionless susceptibility $\hat{\chi}$ by

$$\hat{\boldsymbol{P}}_{\text{eff}} = \varepsilon_1 V \hat{\chi} \hat{\boldsymbol{E}} \tag{3}$$

enables us to put the electric torque into the simple form

$$T_e = -\varepsilon_1 V E^2 \,\mathrm{Im}\,\hat{\chi}. \tag{4}$$

As the rotating electric field (2) can be written as the sum of two orthogonal sinusoidal fields with the same frequency and with $\pi/2$ phase shift, the susceptibility is identical to that found for alternating fields:

$$\hat{\chi} = [K + N/(1 - \mathrm{i}\omega\tau)] \tag{5}$$

where

$$\begin{aligned} K &= 3(\varepsilon_r - 1)/(\varepsilon_r + 2) \\ N &= -9(\varepsilon_r - \sigma_r)/(\varepsilon_r + 2)(\sigma_r + 2). \end{aligned} \tag{6}$$

From (4)-(6) the electric torque can be expressed as

$$T_e = -\varepsilon_1 V N E^2 x/(1 + x^2) \tag{7}$$

where $x = \omega\tau$ and we have changed back to the frame at rest with respect to the field by the substitution $\omega \to -\omega$.

Taking into account the viscous torque

$$T_\eta = -6V\eta\omega \tag{8}$$

η being the dynamical viscosity of the external medium, the equation of motion of the sphere,

$$I\,\mathrm{d}\omega/\mathrm{d}t = T_e + T_\eta \tag{9}$$

can be put into the following dimensionless compact form (Turcu 1987):

$$\frac{\mathrm{d}x}{\mathrm{d}t} = cx\left(\frac{p}{1+x^2} - 1\right) \tag{10}$$

where

$$c = \frac{6V\eta}{I} \qquad p = \frac{E^2}{E_c^2}\,\mathrm{sgn}(\varepsilon_r - \sigma_r) \qquad E_c^2 = \frac{2\eta\sigma_1(\sigma_r+2)^2}{2\varepsilon_1^2|\varepsilon_r - \sigma_r|} \tag{11}$$

and I is the momentum of inertia of the sphere.

For small values of the field ($p < 1$) equation (10) admits only the trivial solution $x_1 = 0$. The picture changes qualitatively if the field strength exceeds the threshold value E_c (i.e. $E > E_c$) when two new real solutions appear. The new solutions are symmetrical and describe the rotational motion that can occur to the right or to the left, the direction of rotation being dictated by chance. The angular frequency of the rotational motion is controlled only by the relative field strength

$$x_{2,3} = \pm(E^2/E_c^2 - 1)^{1/2}. \tag{12}$$

It must be emphasised that the appearance of the spontaneous rotation is ensured only if $\varepsilon_r > \sigma_r$, otherwise $p < 0$ and the static steady state remains stable.

Let us consider now the model of the spherical shell. The whole reasoning remains unchanged, only the dimensionless susceptibility must be rewritten as

$$\hat{\chi} = K + N_1/(1+\mathrm{i}\omega\tau_1) + N_2/(1+\mathrm{i}\omega\tau_2) \tag{13}$$

where

$$\begin{aligned} K &= 3(\varepsilon_r - 1)/(\varepsilon_r + 2) \\ N_1 &= -[9\sigma_r/2(\sigma_r+2)][1 + \sigma_{rm}(\sigma_r+2)/2\delta\sigma_r]^{-1} \\ N_2 &= -9(\varepsilon_r - \sigma_r)/(\varepsilon_r+2)(\sigma_r+2). \end{aligned} \tag{14}$$

We are interested once again in the steady state solutions of the equation of motion which, for the spherical shell model, becomes

$$\mathrm{d}y/\mathrm{d}t = cy[p_1/(1+y^2) + ap_2/(1+a^2y^2) - 1] \tag{15}$$

where

$$\begin{aligned} &y = \omega\tau_1 && a = \tau_2/\tau_1 \\ &p_1 = E^2/E_{c_1}^2 && p_2 = (E^2/E_{c_2}^2)\,\mathrm{sgn}(\varepsilon_r - \sigma_r) \\ &E_{c_j}^2 = \frac{2\eta}{\varepsilon_1\tau_1|N_j|} && j = 1, 2. \end{aligned} \tag{16}$$

In the case of thin shells at low conductivity, conditions largely accomplished by biological cells for instance, the inequality

$$a \ll 1 \tag{17}$$

allows us to neglect, in a first approximation, the second term of the right-hand side of (15). Consequently one obtains results formally identical with those of the spherical case. There are nevertheless two main points that must be noted.

(i) The sign of the dimensionless parameter p_1 is always positive so that the spinning always begins as soon as the electric field strength exceeds the threshold value E_{c_1}.

(ii) Because $\tau_1 \gg \tau_2$ the relative contribution of the second term of the right-hand side of (15) is drastically diminished and the spontaneous dynamical symmetry breaking is controlled by the membrane contribution to the induced effective dipole moment.

It must also be emphasised that the threshold value of the electric field strength is diminished with respect to the spherical case:

$$E_{c_1}^2/E_c^2 = a|N_2|/|N_1| \ll 1. \tag{18}$$

As a direct consequence, the rotation of small spherical bodies is much easier to accomplish if they are surrounded by a thin shell of low conductivity.

3. Electrorotation in alternating homogeneous fields

Electrorotation can occur not only in static electric fields but can also appear in a homogeneous alternating electric field.

The problem of the spontaneous rotation of a single sphere subjected to an alternating electric field was analysed by one of us in a previous paper (Turcu 1987). Essentially the novelty introduced by the extension from static to sinusoidal fields consists of the appearance of a domain in the bidimensional parameter space (having the field strength and the field frequency as coordinates) where several competing dynamics are locally stable. The static steady state, for instance, has a metastable behaviour before becoming unstable. The rotational motion develops discontinuously via a non-stationary regime in contrast to the static electric field case when the spinning develops continuously from rest.

The results can be extended to the spherical shell model by adding to the effective dipole moment a new term representing the shell contribution. In the limit $\tau_1 \gg \tau_2$ the shell contribution becomes dominant, so that all the results of Turcu (1987) are reproduced. The only thing that must be done is to replace the dimensionless parameter p by p_1.

It must be noted that the rich dynamical picture emerging from the introduction of the electric field frequency as a new parameter is, in the spherical shell model, mostly determined by the shell properties.

4. Electrorotation in rotating electric fields

From a physical point of view the electrorotation can be induced in two ways.

(i) By raising the electric field strength above a threshold value at which point a spontaneous dynamical symmetry breaking can occur and a field-dependent rotational motion can be obtained.

(ii) By breaking from outside the axial symmetry by subjecting a suspended body to a rotating electric field with variable angular frequency.

In the present section we shall investigate the second line of action for both the spherical and the spherical-shell models.

4.1. *The spherical model*

As we have seen in the previous section, when a sphere is subjected to a rotating electric field

$$\hat{\boldsymbol{E}} = (\boldsymbol{e}_1 - \mathrm{i}\boldsymbol{e}_2)\exp(\mathrm{i}\omega_0 t) \tag{19}$$

the standing sphere response is characterised by the dimensionless susceptibility

$$\hat{\chi} = K + N/(1 + \mathrm{i}\omega_0\tau). \tag{20}$$

By taking the imaginary part of (20) one finds the expression for the electric torque

$$T_\mathrm{e} = 3V\varepsilon_1 NE^2 x_0/(1 + x_0^2) \tag{21}$$

where $x_0 = \omega_0\tau$.

When the sphere rotates with an angular frequency ω we must, replace ω_0 in (21) by the difference $\omega_0 - \omega$. Taking into account also the torque imparted by the viscous forces, the equation of motion has the following compact form:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = c\left(\frac{p(x - x_0)}{1 + (x - x_0)^2} - x\right). \tag{22}$$

One can remark that when $x_0 = 0$, equation (22) is identical to (10) valid for static electric fields.

The steady state solutions of (22) are obtained as the real roots of the third-order algebraic equation

$$x[1 + (x - x_0)^2] = p(x - x_0). \tag{23}$$

It must be emphasised that in contrast to the case of an electrostatic field the static steady state is not admitted as a solution by the equation of motion for a particle subjected to rotating electric fields. The third-order equation (23) has always at least one real non-zero solution for arbitrary small field strengths. Consequently there is no threshold value for the field.

For field strengths smaller than the threshold value $E_\mathrm{c}(p < 1)$ equation (23) has only one real solution

$$x_1 = [A + (A^2 - B^3)^{1/2}]^{1/3} + [A - (A^2 - B^3)^{1/2}]^{1/3} \tag{24}$$

where

$$A = -\tfrac{1}{54}[2x_0^3 + 9x_0(p + 2)] \qquad B = \tfrac{1}{3}[x_0^2 + 3(p - 1)]. \tag{25}$$

In the limit of very small values of $p(p \ll 1)$ it is not difficult to observe directly from (22) that the solution x_1 also satisfies the inequality $x_1 \ll 1$ so that it can be very well approximated by

$$x_1 = -px_0/(1 + x_0^2). \tag{26}$$

This is the expression usually found in the literature (Arnold and Zimmermann 1982, Fuhr *et al* 1986).

It is useful to remark that the dimensionless electrorotation frequency can be simply expressed in terms of the imaginary part of the susceptibility

$$x_1 = -(\varepsilon_1\tau E^2/6\eta)\,\mathrm{Im}\,\hat{\chi} \tag{27}$$

having the same dependence on the angular frequency of the rotating electric field. The shape of the electrorotation spectra for several values of the parameters ε_r and σ_r are shown in figure 1.

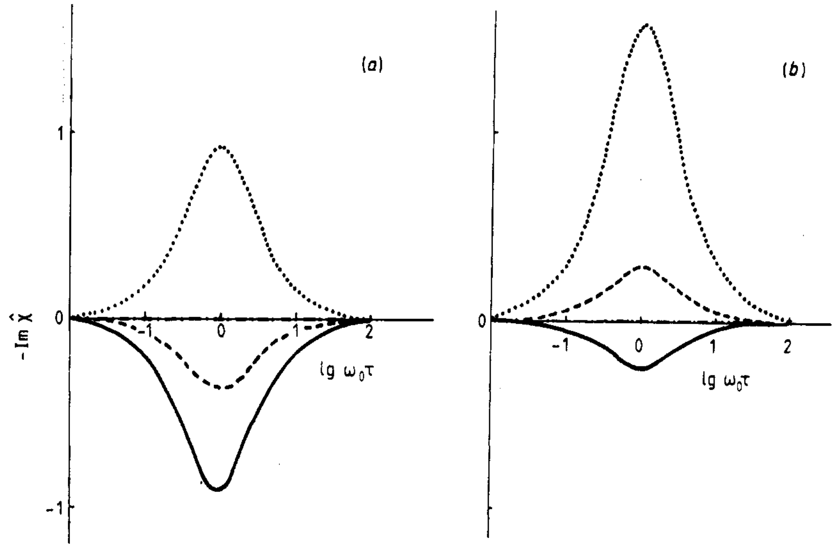

Figure 1. Frequency dependence of the imaginary part of the complex electric susceptibility in the spherical model for (*a*) $\varepsilon_r = 2$ and (*b*) $\varepsilon_r = 0.5$ and for several values of the dimensionless parameter $\sigma_r = 20$ (dotted curve), 1 (broken curve), ε_r (chain curve), 0.2 (full curve).

As the coefficient N may be either positive or negative the electrorotation in rotating electric fields can be co-field or counter-field, with $\varepsilon_r > \sigma_r$ or $\varepsilon_r < \sigma_r$ respectively.

For sufficiently strong fields such that $E > E_c\,(p > 1)$ all the roots can be real, and have the following expressions:

$$x_1 = \tfrac{2}{3}x_0 + 2\sqrt{B}\cos(\zeta/3)$$
$$x_{2,3} = \tfrac{2}{3}x_0 - 2\sqrt{B}\cos[(\zeta \pm 2\pi)/3] \qquad (28)$$

where

$$\cos\zeta = A/\sqrt{B^3}. \qquad (29)$$

Nevertheless it must be emphasised that the last two roots become rapidly complex when the field angular frequency ω_0 increases. For a given value of $p(p > 1)$, roots $x_{2,3}$ are real only in the restricted domain

$$A^2 < B^3. \qquad (30)$$

The appearance of some competitive dynamics in the low-frequency range, as soon as the field strength exceeds a threshold value, is due to the competition between the two different ways of inducing the rotational motion. As can be seen, the field-induced electrorotation is effective only for static or slowly rotating strong electric fields.

The electrorotation can be induced at much smaller values of the electric field strength if continuous (Arnold and Zimmermann 1982) or pulsed (Mischel *et al* 1982) rotating fields with variable frequency up to 1 MHz, for instance, are used.

Experimentally measured electrorotation spectra (Mischel and Pohl 1983, Glaser *et al* 1983, Lovelace *et al* 1984) are generally well fitted by the formula (26).

By extending the frequency of the external rotating field to higher values, a second maximum was found (Fuhr *et al* 1985) at a higher characteristic frequency. Only the spherical-shell model predicts such a behaviour, as has been shown by Fuhr *et al* (1986).

4.2. *The spherical-shell model*

For the spherical shell, the susceptibility characterising the response to the rotating electric field (19) is given by

$$\hat{\chi} = K + N_1/(1+\mathrm{i}\omega_0\tau_1) + N_2/(1+\mathrm{i}\omega_0\tau_2) \tag{31}$$

from which the expression for the electric torque is found to be

$$T_e = \varepsilon_1 VE^2\left(\frac{N_1\omega_0\tau_1}{1+\omega_0^2\tau_1^2}+\frac{N_2\omega_0\tau_2}{1+\omega_0^2\tau_2^2}\right). \tag{32}$$

Remembering that N_1 and N_2 are given by (14) it must be noticed that the shell contribution given by the first term is always negative in contrast to the second term which can have both signs.

An expression having a similar form was earlier obtained by Fuhr *et al* (1986) but in our opinion some error has been made, such that their formula (12) gives wrong results in the two limiting cases $\delta \to 0$ or $\hat{\varepsilon}_m \to \hat{\varepsilon}_2$ when the simple spherical model must be recovered.

The second limit is not directly applicable to (14) and (32) on account of the approximations that have been made but the general expressions (19), (22) and (27) from our preceding paper have the right limits.

With the formula (32) modified in order to take into account the rotation of the sphere, one obtains the equation of motion for a spherical shell

$$\frac{\mathrm{d}y}{\mathrm{d}t} = c\left[\frac{p_1(y-y_0)}{1+(y-y_0)^2}+\frac{ap_2(y-y_0)}{1+a^2(y-y_0)^2}\right] \tag{33}$$

where

$$y = \omega\tau_1 \qquad y_0 = \omega_0\tau_1. \tag{34}$$

As for the spherical model, we are interested in the steady state solutions.

For small field strengths ($p_{1,2} \ll 1$) there is again only one real solution given with a very good approximation by

$$y_1 = -p_1y_0/(1+y_0^2) - p_2ay_0/(1+a^2y_0^2) \tag{35}$$

which can be put into the compact form

$$y_1 = -(\varepsilon_1\tau_1E^2/6\eta)\,\mathrm{Im}\,\hat{\chi}. \tag{36}$$

Several typical electrorotation spectra calculated for the same parameters as for the spherical model are shown in figure 2.

As one can see, the spectra remain almost unchanged in the high-frequency range. The contributions introduced by the shell consist of the appearance of a second characteristic frequency located in the lower-frequency range and of a corresponding maximum in the electrorotation spectra. As for the simple spherical model the electrorotation can have a co-field or a counter-field direction for high frequencies but always has a counter-field direction in the lower-frequency range.

For sufficiently strong fields such that $E > E_{c_1}(p_1 > 1)$ and for small values of the rotating field frequency equation (33) has three real roots. As $a \ll 1$ the contribution of the second term in the right-hand side of (33) can be omitted and an equation similar to (22) is obtained. Consequently all that we have said in the case of the spherical model becomes valid; the only thing that must be done is to replace p, x and x_0 by p_1, y and y_0 respectively in the formulae (25), (28)–(30).

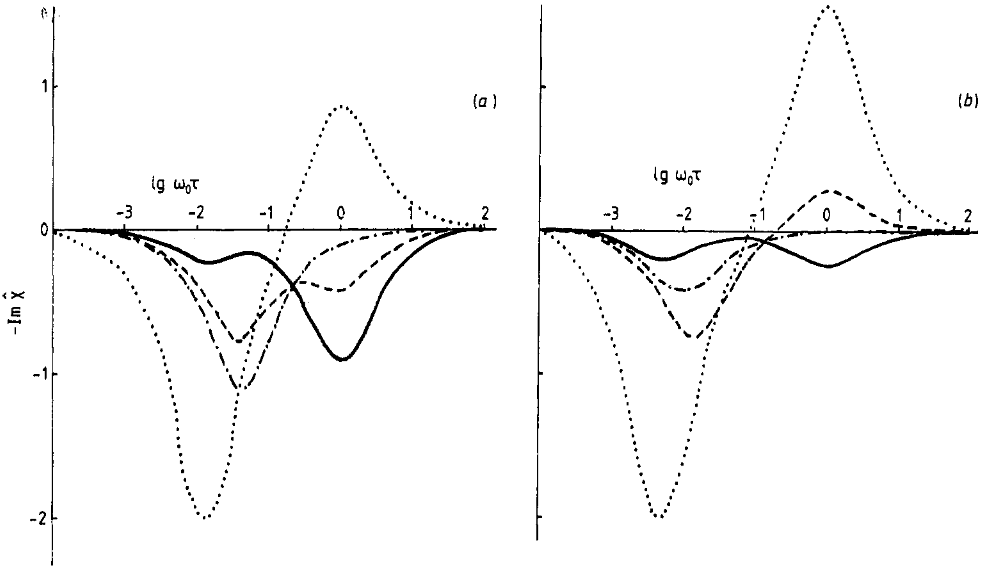

Figure 2. Frequency dependence of the imaginary part of the complex electric susceptibility in the spherical shell model for $\sigma_{rm} = 0$, $\delta = 10^{-3}$, $\varepsilon_{rm} = 0.1$, and for (*a*) $\varepsilon_r = 2$ and (*b*) $\varepsilon_r = 0.5$, and for several values of $\sigma_r = 20$ (dotted curve), 1 (broken curve), ε_r (chain curve), 0.2 (full curve).

5. Conclusions

The electrorotation of small physical objects suspended in a conducting dielectric medium can be induced in two different ways.

(i) By a spontaneous dynamical symmetry breaking mechanism using static or alternating electric fields.

(ii) By breaking from outside the axial symmetry by subjecting the suspended body to a rotating electric field with variable frequency.

The first possibility requires high values for the electric field strength, the rotational motion appearing only if the field exceeds a threshold. By examining the possibility of inducing this type of rotation for a simple spherical particle and for a spherical particle surrounded by a single thin shell at low conductivity we can conclude that the presence of the shell can drastically reduce the threshold value for the electric field strength.

The second way seems to be much easier when the rotational motion can be induced even for small values of the electric fields.

By comparing the electrorotation spectra obtained in the spherical and in the spherical-shell models one can see that the shell brings a second maximum located at a new characteristic frequency. For thin shells of low conductivity this frequency is smaller than the characteristic frequency for the spherical model so that once again the presence of the shell seems to be useful.

We want to emphasise that all the electrorotation phenomena were described in terms of the imaginary part of the electric susceptibility calculated for a Maxwell-Wagner interfacial polarisation mechanism. In the previous paper we analysed the dielectrophoretic spectra for both the above-mentioned models. The dielectrophoretic force is proportional to the real part of the same electric susceptibility. By examining now the dielectrophoretic and the electrorotation spectra one can see that they are strongly interrelated, being controlled by the same parameters and having the same characteristic frequencies. The information obtained from the experimental determination of one type of spectrum is enough to find the complex susceptibility and to completely characterise both phenomena.

After our two papers were written we learned of a recent paper by Pastushenko *et al* (1988) who elaborated a unitary theory of electrorotation and dielectrophoresis. However, our treatment covers a wider range of phenomena and in addition offers some useful analytical expressions.

References

Arnold W M and Zimmermann U 1982 *Z. Naturf.* **37c** 908–15
Fuhr G, Glaser R and Hagedorn R 1986 *Biophys. J.* **49** 395–402
Fuhr G, Hagedorn R and Müller T 1985 *Stud. Biophys.* **107** 109–16
Glaser R, Fuhr G and Gimsa J 1983 *Stud. Biophys.* **96** 11–20
Hagedorn R and Fuhr G 1984 *Stud. Biophys.* **102** 229–38
Holzapfel C, Vienken J and Zimmermann U 1982 *J. Membrane Biol.* **67** 13–26
Jones T B 1984 *IEEE Trans. Ind. Appl.* **IA-20** 845–9
Lampa A S 1906 Sitzungsberichte dk Akademie der Wissenschaften **115** 1659–90
Lovelace R V E, Stout D G and Steponkus P L 1984 *J. Membrane Biol.* **82** 157–66
Mischel M and Pohl H A 1983 *J. Biol. Phys.* **11** 98–112
Mischel M, Voss A and Pohl H A 1982 *J. Biol. Phys.* **10** 223–6
Pastushenko V Ph, Kuzmin P I and Chizmadzhev Yu A 1988 *Biol. Membrany* **5** 615–78
Pohl H A and Crane J S 1971 *Biophys. J.* **11** 711–27
Quincke G 1896 *Ann. Phys. Chem.* **59** 417–86
Sauer F A and Schlögl R W 1985 *Interaction Between Electromagnetic Fields and Cells* (New York: Plenum)
Teixeira-Pinto A A, Nejelski L L, Cutler J L and Heller J H 1960 *Exp. Cell. Res.* **20** 548–64
Turcu I 1987 *J. Phys. A: Math. Gen.* **20** 3301–7
Turcu I and Lucaciu C M 1989 *J. Phys. A: Math. Gen.* **22** 985